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On the von Neumann paradox of weak Mach reflection *

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Abstract. Recent experimental and numerical studies of weak Mach reflections are examined. It is shown that the fundamental reason for the von Neumann paradox is that his theory of Mach reflection is based on the assumption that the flow downstream of the reflected wave and the Mach shock near the wave triple point is uniform. The assumption is shown to be valid for strong Mach reflection which agrees with experiment, but invalid for weak Mach reflection which does not agree with experiment. It is also shown that viscous effects are dominant when the incident shock is within about 100 mean free path lengths of the corner, but not otherwise. The analytical theory of the entire subsonic region supports these conclusions.

1. Introduction

The first systematic study of the reflection of shock waves at rigid surfaces was carried out by von Neumann in 1943. He developed the theory for a compressible substance with an arbitrary equation of state, but gave most attention to air which he considered to be a perfect gas, and some attention to water-like substances. For the perfect gas, he was able to deduce specific algebraic equations which solved the problems of both regular reflection, RR, and Mach reflection, MR. the theory of Mach reflection was based upon the Rankine–Hugoniot, R–H, jump conditions and on the following boundary conditions at the shock triple point (fig. 1):

$$p_1 - p_2 = 0, \tag{1}$$

$$\delta' + \delta_2 - \delta_1 = 0. \tag{2}$$

* Results based on conclusions made at the "Workshop on weak Mach reflection", March 31 – April 1, '88 at Tokyo Denki University.

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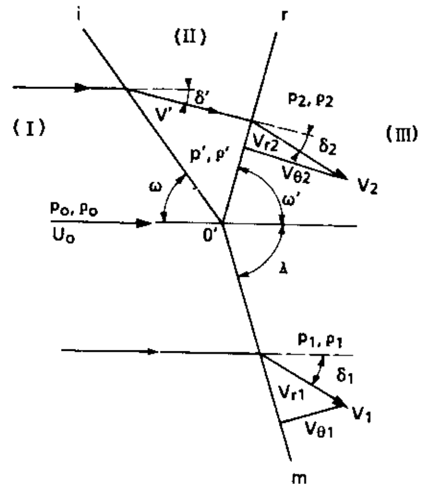


Fig. 1. Three-shock configuration.

The R–H conditions were also used for the theory of RR, but the boundary condition was then merely

$$\delta' + \delta_2 = 0. \tag{3}$$

Von Neumann's theory showed that a solution, or set of solutions, for RR or MR was determined once the following set of parameters was given (γ, ξ, θ), where γ is the ratio of specific heats, $\xi = p_0/p'$, is the inverse strength of the incident shock i, and θ is, say, the corner angle of the disturbing boundary (fig. 2a). It also showed that for a given medium (γ), transition between regular and Mach reflection, denoted by RR $\leftrightarrows$ MR, could be induced by a continuous change in one or more of the system parameters ξ or θ. When he studied the conditions that would enable him the transition, he concluded that the criterion for transition depended on whether i was a weak or a strong shock. He decided that transition occurred at the detachment point for weak shocks and at the normal shock point – also called the mechanical equilibrium point or the von Neumann point – for strong shocks. He gave a rigorous definition of strong and weak shocks which was based upon a geometric property of the polar diagram. A more convenient definition is that i is weak when the flow downstream of the reflected shock, r, in Mach reflection is subsonic, $M_2 < 1$, and that i is strong when it is supersonic, $M_2 > 1$. This means of course that the weak/strong boundary is at the sonic condition, $M_2 = 1$. The two definitions differ slightly in the critical value (Henderson, 1987) they give for $\xi = \xi_{cr}$.

Subsequently experiments were done to compare with the theory. The agreement was satisfactory for strong and weak regular reflection, and also for strong Mach reflection, but there were large discrepancies for weak Mach reflection as displayed in fig. 2b (Bleakney and Taub, 1949) and fig. 2c (Henderson and Siegenthaler, 1980). Indeed, it was even found that for some values of (γ, ξ, θ), the theory had no physically acceptable solutions, and yet experiment showed nevertheless that a Mach reflection – or something very much alike existed. This rather remarkable discrepancy was dubbed the "von Neumann paradox" by Birkhoff (1950). It is difficult to escape the conclusion that the von Neumann model of weak Mach reflection is inadequate.

In the forty years or so since its discovery the paradox has attracted many researchers, and their work has ranged over many analytical and experimental studies (see, for example, list of references in Henderson (1987), also Hornung (1986)). In spite of this impressive amount of work, there is still no general agreement that the problem has been solved. In recent years

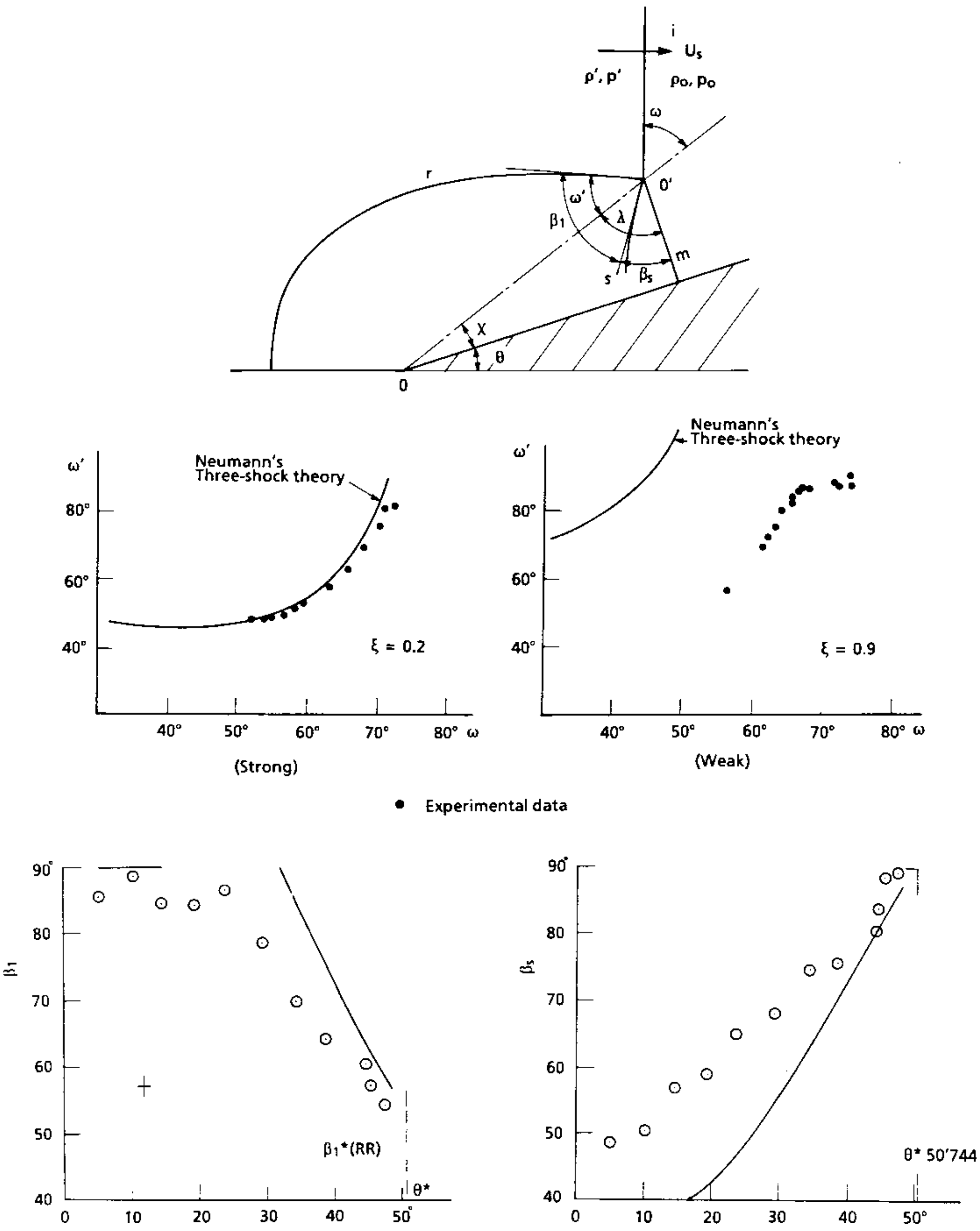


Fig. 2. (top) Incident shock wave (i) passing over a concave corner; (middle) Display of "Neumann paradox": comparison of Neumann's three-shock theory with experimental dada (Bleakney and Taub, 1949) for strong ($\xi = 0.2$) and weak ($\xi = 0.9$) cases; (bottom) Comparison of the shock wave angles (β_s, β_1 given by the Neumann theory with their experimental data at the triple point.

however, the climate of research has changed dramatically with improvements both in experimental techniques and in numerical algorithms for use with super computers.

In the present paper, we review some of the recent progress with the objective of improving the understanding of the physics of weak Mach reflection and the paradox based on the above progress as well as analytical consideration. This should form the basis for a sounder physical model for the phenomenon.

2. Recent results

2.1. Experiment

Optical methods have been of special importance for providing information on Mach reflection. Shadowgraph and schlieren techniques have enabled measurements of wave angles at

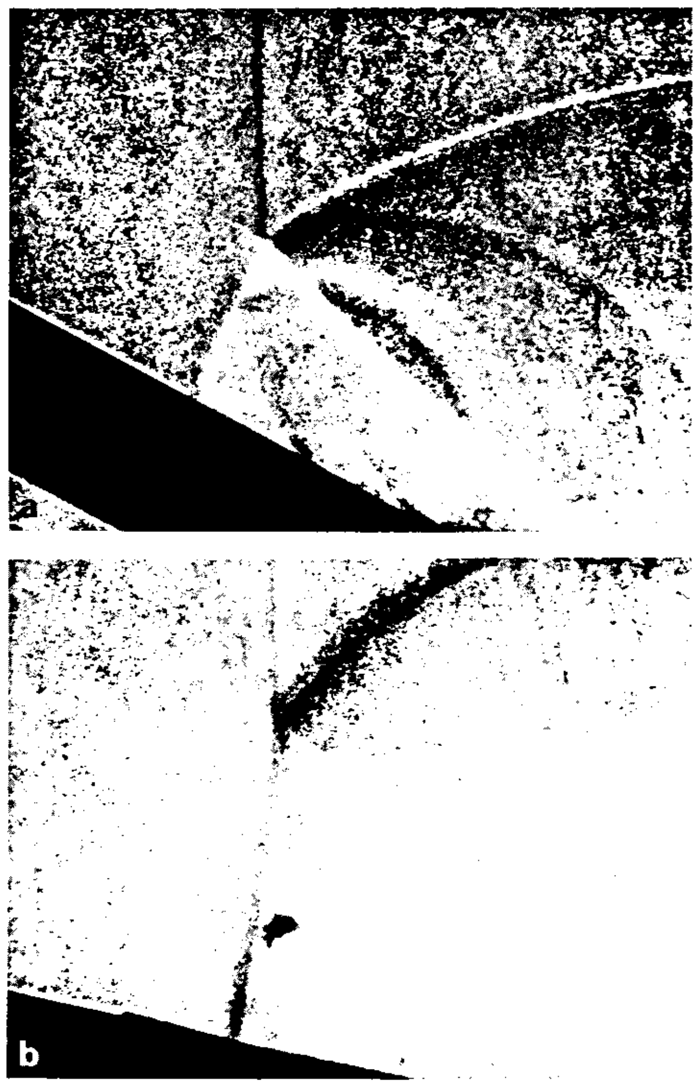
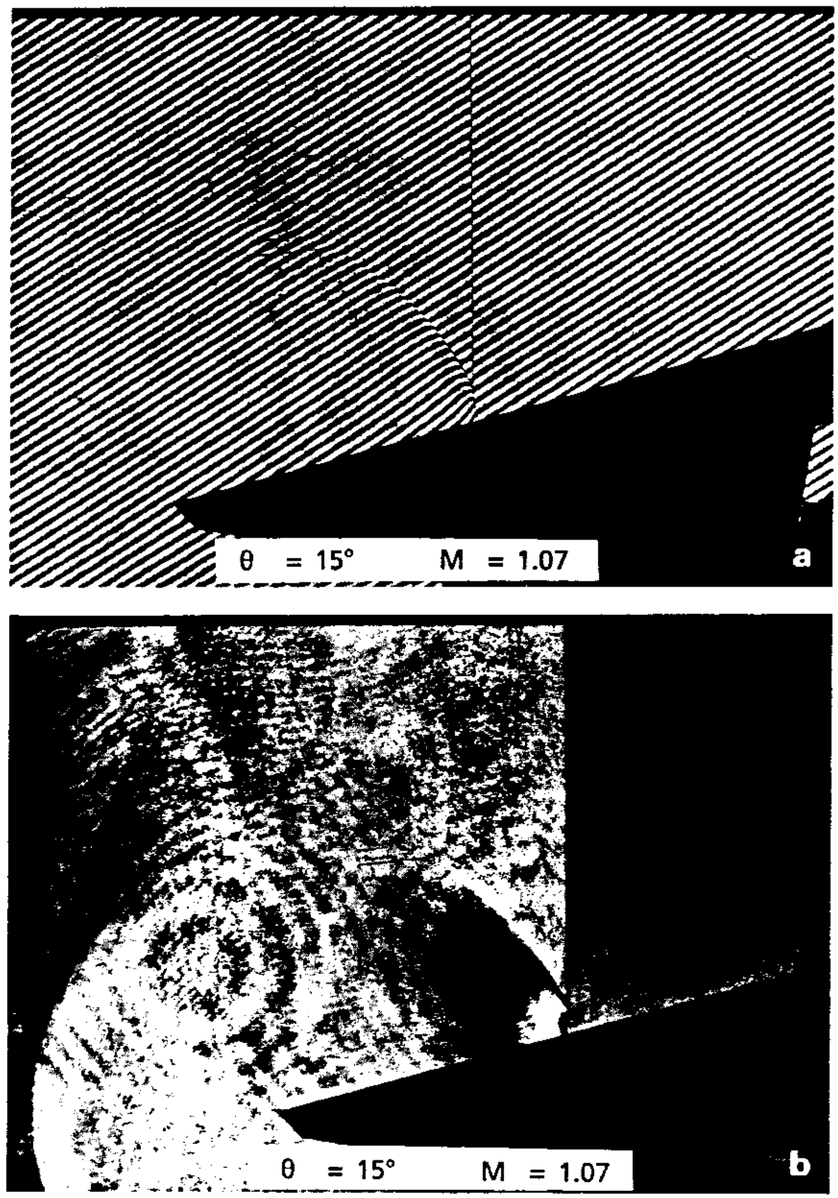


Fig. 3. (a,b) Examples of schlieren photographs of weak Mach reflection (Henderson and Woolmington, 1985).

the triple point, and also measurements of certain shock wave Mach numbers, such as that of the Mach shock along the ramp surface. However, with the advent of the laser it is now comparatively easy to construct a holographic interferometer and thereby obtain quantitative information not only at particular points, but for the entire flow field, since the interferometer gives isopycnic lines. Examples of schlieren photographs of weak Mach reflection are shown in fig. 3a,b (Henderson and Woolmington, 1985) and of interferometric photographs in fig. 4a,b (Takayama, 1988).

For certain values of (γ, ξ, θ) the von Neumann theory does provide physically acceptable solutions. Not surprisingly, perhaps, these occur near the weak/strong boundary. The theory



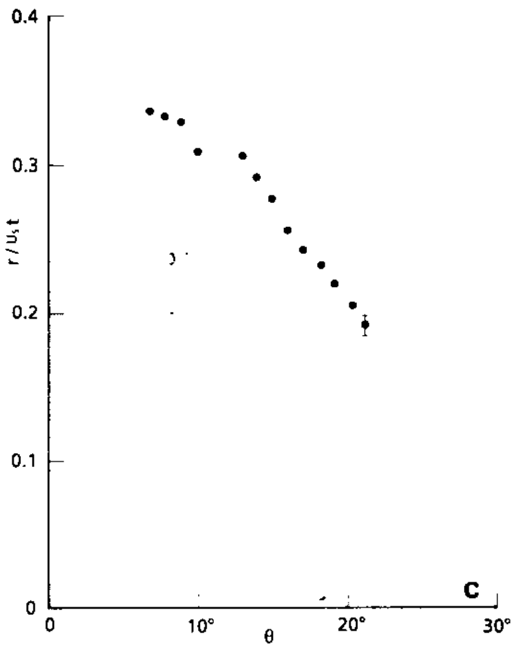


Fig. 4. (a,b) Examples of interferometer photographs of weak Mach reflection ($M = 1.07$); (c) Minimum radius of curvature r of Mach shock wave (m) measured from photographs such as above for various θ angles (Takayama, 1988).

makes it possible to calculate wave angles at the triple point for comparison with experiment (fig. 2b,c). The discrepancy between theory and experiment is mostly much larger than the experimental error, so it must be concluded that the theory is incorrect in some way.

Further away from the weak/strong boundary there exist regions of (γ, ξ, θ) for which the theory gives no physically acceptable solutions, and yet experiment shows that nevertheless a Mach reflection – or something like it – does in fact exist (fig. 3b and 4a,b). In this case it must be concluded that the theory is a complete failure.

Fig. 3a shows a photograph of weak Mach reflection for which the theory does provide a solution (although it does not agree well with experiment), and fig. 3b shows one for which there is no solution. Perhaps the most noticeable difference between these is that the Mach shock shows a stronger curvature near the triple point in fig. 3a. In fig. 3b the incident and Mach shocks appear to be part of a single shock with a continuously curving tangent; this is even more noticeable in fig. 4a,b where the incident shocks are even weaker. In this case it is possible to measure the minimum radius of curvature at, or near, the triple points. Some results are shown in fig. 4c, and indicate that the radius of curvature is finite and conversely the curvature also.

This also suggests that the thickness of the reflected wave is by no means negligible. These facts are contrary to some of the assumptions of the von Neumann theory.

2.2. *Effects of viscosity*

Experiments have been carried out in both dense and in very low density gases near the slip flow regime. It has been noted that the theory is in good agreement with experiment for strong Mach reflection, but it is based on the assumption that the gas is inviscid, so it must be concluded that viscous effects are negligible for the flow near the triple point. For the weak case the flow downstream is subsonic by definition, so that the entire field can affect the flow near the triple point (Sakurai, 1964). In particlular the wall boundary layer can have an

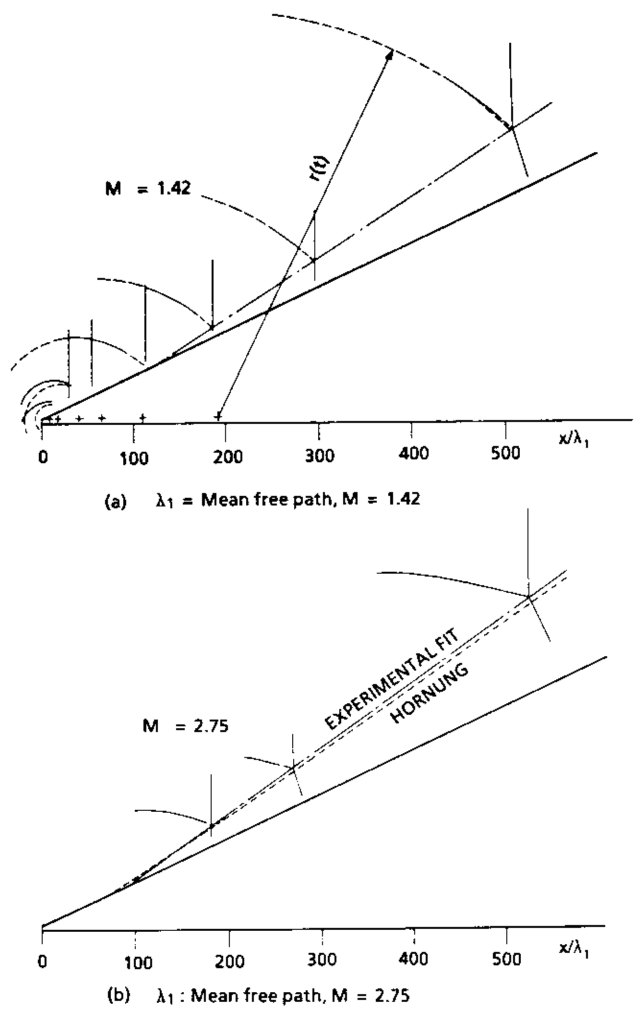


Fig. 5. Low density gas experiments (Walenta, 1983, 1987).

influence; for example in shock tube experiments there is a negative displacement effect which can modify the trajectory path of the triple point, with the consequence that the triple point does not pass through the corner (Henderson and Gray, 1981; Reichenbach, 1985). Initially, when the incident shock first encounters the corner, there may also be a local strong viscous effect, which experiment indicates rapidly decays.

In the low density gas experiments of Walenta (1983, 1987), it was found that it took a significant time for the reflected wave to form, and that it was substantially thicker than the incident shock. The initial reflection off the sloping surface appeared to be of the regular type, but after about hundred mean free paths from the corner a Mach reflection appeared. In other words, here also the trajectory path of the triple point did not pass through the corner (fig. 5). This is also an effect of viscosity.

2.3. Numerical methods

Numerical simulations of strong Mach reflection have been successful in that they do agree in detail with experiments, for example those of Glaz et al. (1985) and Hornung (1986).

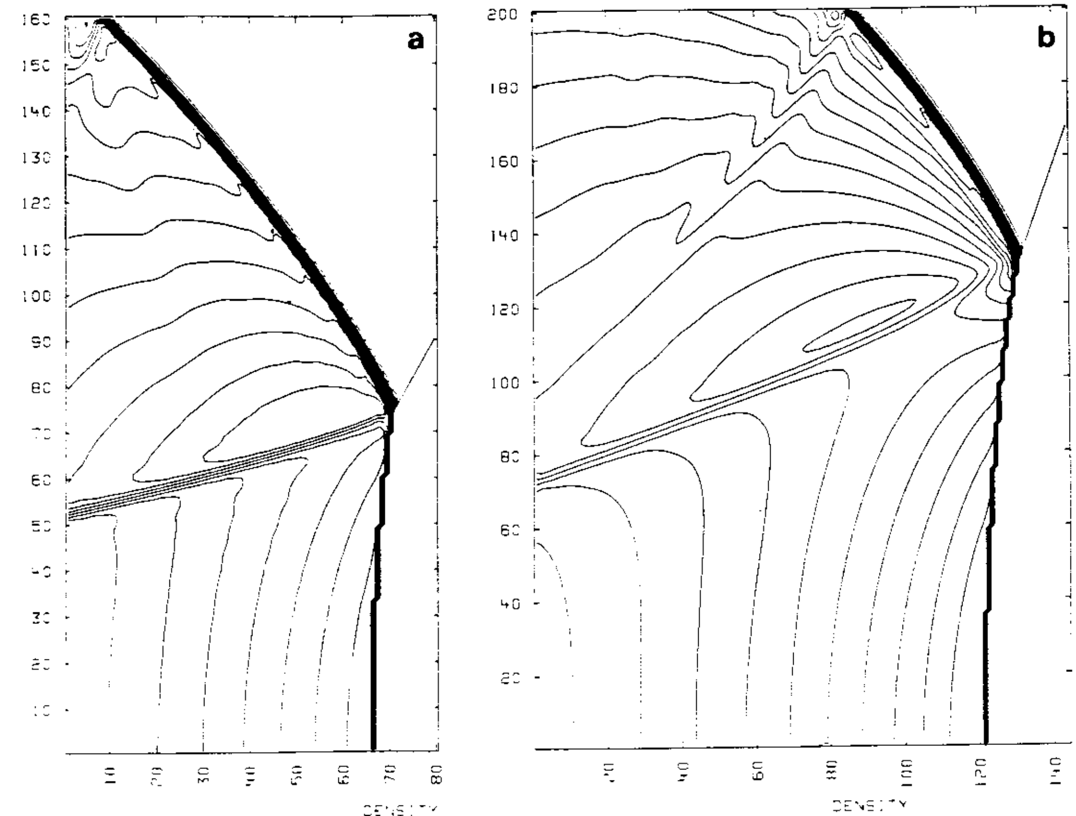


Fig. 6. Results of numerical simulation (Colella and Henderson, Density distribution, 1988); (a) $\gamma = 5/3$, $\xi = 0.406$, $\theta = 29.27°$ for which the Neumann theory gave a solution; (b) $\gamma = 5/3$, $\xi = 0.414$, $\theta = 19.30°$ for which the Neumann theory did not give a solution.

However, no successful computations for weak Mach reflections have yet been published. Recently Colella and Henderson (1988) have attacked the problem using a second-order Godunov code for inviscid fluid. They calculated the entire flow field for values of (γ, ξ, θ) for which the von Neumann theory gave a solution and those for which it did not; see fig. 6 for example (a), for (5/3, 0.406, 29.27°), and (b), (5/3, 0.414, 19.30°). They then obtained the trajectory path angle χ of the wave triple point and the Mach number of M_n of the Mach shock along the ramp and compared these data with their experiments in argon. These results are shown in fig. 7a for $\gamma = 5/3$, $\langle \xi \rangle = 0.406$, and fig. 7b for even weaker shocks, $\gamma = 5/3$, $\langle \xi \rangle = 0.919 \cdot$. This is a more robust comparison with experiment than comparing wave angles as in fig. 2b, since the waves are curved at the triple point in the weak case and there is some uncertainty where to draw the tangents to make the measurements; there is no such difficulty with (χ, M_n).

3. Discussion

3.1. Possible reason for the failure of the von Neumann theory of weak Mach reflection

Reasons for the failure of the von Neumann theory of weak Mach flow can include the effects of viscosity, unsteadiness, nonlinearity, and nonuniformities in the flow. Experiment

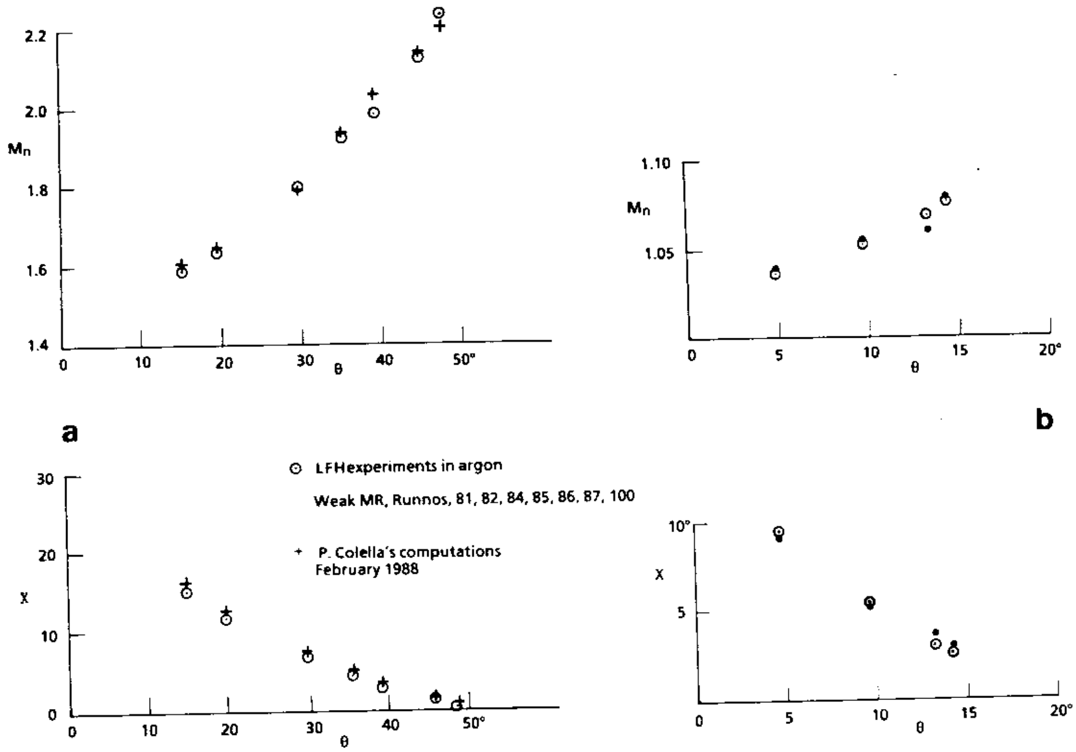


Fig. 7. Trajectory path angle χ of the triple point and the Mach number M_n of the Mach shock obtained from the numerical simulation compared with experimental data in argon; (a) for $\gamma = 5/3$, $\langle \xi \rangle = 0.406$; (b) $\gamma = 5/3$, $\langle \xi \rangle = 0.919$.

shows that viscosity is not dominant except near time $t = 0$, where the incident shock first encounters the corner. Near this condition the experiments of Walenta (1983, 1987) indicate that it is then dominant. However, at longer times the effect of viscosity is small at high Reynolds number and due mainly to the wall boundary layer. This conclusion is also supported by a transformation of the Navier–Stokes equations as given by Sakurai (1985). The salient features are as follows.

Take the x, y, and the origin O as the apex of the wedge. Let $\boldsymbol{v} = (u, v)$ be the velocity, p the pressure, ρ the density, and T the temperature of the flow field. The Navier–Stokes equations of continuity, momentum and energy for two-dimensional, viscous, heat conducting, ideal gas flow are

$$
\begin{aligned}
&\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho \operatorname{div} \boldsymbol{v} = 0, \\
&\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\operatorname{grad} p + \frac{\mu}{3} \operatorname{grad} \operatorname{div} \boldsymbol{v} + \mu \Delta \boldsymbol{v}, \\
&\frac{p}{\gamma - 1} \frac{\mathrm{D}}{\mathrm{D}t} [\log(p \rho^{-\gamma})] \\
&\quad = \mu \left[2 \left\{ \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right\} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 - \tfrac{2}{3} (\operatorname{div} \boldsymbol{v})^2 \right] + \kappa \, \Delta T,
\end{aligned}
\tag{4}
$$

where κ, μ, are the coefficients of heat conduction and viscosity, respectively.

Introduce the dimensionless similarity variables X, Y defined by

$$X = \frac{x}{ct}, \quad Y = \frac{y}{ct}, \quad U = \frac{u}{c} - X, \quad V = \frac{v}{c} - Y, \quad P = \frac{p}{p_1}, \quad R = \frac{\rho}{\rho_1} \tag{5}$$

and using

$$s = \frac{\mu}{p_1 t},$$

where p_1, ρ_1 are the pressure and the density values of a certain uniform state, and $c^2 = \gamma p_1/\rho_1$.

Then the system of equations (4) is transformed into

$$\begin{aligned}
&UR_X + VR_Y + R(U_X + V_Y + 2) = sR_s,\\
&R(UU_X + VU_Y + U) + \frac{1}{\gamma}P_X = s\left[RU_s + \tfrac{1}{3}(U_X + V_Y)_X + U_{XX} + U_{YY}\right],\\
&R(UV_X + VV_Y + V) + \frac{1}{\gamma}P_Y = s\left[RU_s + \tfrac{1}{3}(U_X + V_Y)_Y + V_{XX} + V_{YY}\right],\\
&UP_X + VP_Y + \gamma P(U_X + V_Y + 2) = s\Big[P_X + (\gamma-1)2\left\{U_X + 1)^2 + (V_Y + 1)^2\right\}\\
&\qquad + (U_Y + V_X)^2 - \tfrac{2}{3}(U_X + V_Y + 2)^2\\
&\qquad + \frac{\gamma}{\sigma}\left\{\left(\frac{P}{R}\right)_{XX} + \left(\frac{P}{R}\right)_{YY}\right\}\Big],
\end{aligned} \tag{6}$$

where σ is the Prandtl number.

Observe that the system of equations (6) is reduced to equations of self-similar flow by putting $s = 0$, and the deviation from the self-similarity appears through the variable s which depends on time t, viscosity, and thermal conductivity.

One of the assumptions, although usually it is not mentioned explicitly, of the von Neumann theory is that the flow is stationary. Now, in the experiments over a concave corner, the weak Mach reflections are unsteady. However if there is a uniform region of flow in the neighborhood of the triple point, then the local flow can be reduced to the stationary state by an elementary Galilean transformation along the trajectory of the triple point. For strong Mach reflections the flow is supersonic and uniform downstream of the reflected shock, so the theory is applicable; this also means for example that wave angles at the triple point can be determined from purely local conditions.

As may be seen from the experimental and numerical results referred to in section 2, the reflected and Mach waves are curved for weak Mach reflection; consequently the flow is non-uniform near the triple point. This implies that the von Neumann theory cannot be applied, and furthermore the entire subsonic flow field downstream of the reflected shock must be taken into account in order to determine the flow near the triple point.

Attempts have been made by many authors to construct analytical theories of the local region. Most of these were done, directly or indirectly, by adding adjustable extra terms to the right-hand sides of eqs. (1) and (2). These terms are found to be typically very small for weak Mach reflection (Sakurai, 1964). It is also found that such local solutions are very sensitive to the magnitude of the new terms Δp and $\Delta\delta$ on the right-hand sides of (1) and (2), which again implies that a local solution is inadequate.

3.2. Solutions for the entire subsonic region

Although it is essential to have a solution for the entire subsonic regions, it is very difficult to construct one. This is of course due to the nonlinearity of eqs. (4), and of the complicated

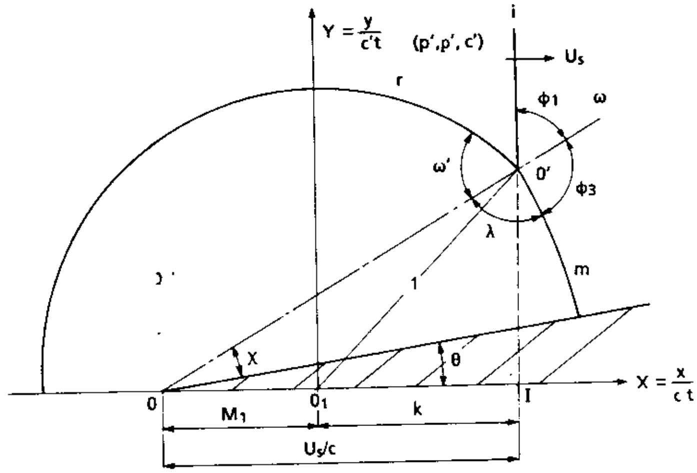


Fig. 8. Geometry of the weak Mach reflection for small wedge angle θ.

boundary conditions that are required. However it is possible to obtain an analytical solution of the linearized version of (4) which is asymptotically valid for small corner angles $\theta \to 0$, (Lighthill, 1948; Sakurai, 1986). Then with θ small, but with the inverse strength ξ of the incident shock i is not necessarily small, the reflected shock is almost reduced to an acoustic wave, and its shape is very nearly cylindrical. Evidently the geometry of the system (fig. 8) gives

$$\tan(\chi + \theta) = \frac{\sqrt{1-k^2}}{M_1 + k},$$

$$\phi_1 + \chi + \theta = \frac{\pi}{2},$$

$$\cos(\pi - (\phi_1 + \omega')) = k,$$

where M_1 is the Mach number of the uniform flow behind the incident shock (i), and $k = \overline{O_1 I}$ and is expressed in terms of ξ as

$$k = \frac{Us}{c_1} - M_1 = \left(\frac{M^2 + \dfrac{2}{\gamma - 1}}{\dfrac{2\gamma}{\gamma - 1} M^2 - 1} \right)^{1/2} = \left(\frac{\gamma + 1}{2\gamma} \xi + \frac{\gamma - 1}{2\gamma} \right)^{1/2},$$

from which we get

$$\chi = -\theta + \tan^{-1} \frac{\sqrt{1-k^2}}{M_1 + k} = -\theta + \tan^{-1} \frac{\sqrt{(1-\xi)\left(\xi + \dfrac{\gamma - 1}{\gamma + 1}\right)}}{1 + \dfrac{\gamma - 1}{\gamma + 1}\xi},$$

$$\phi_1 = \frac{\pi}{2} - (\chi + \theta);$$

$$\omega' = \pi - \cos^{-1} k - \phi_1.$$

Now, the shape of the Mach stem, m, depends on the nature of the flow field, and it must be determined by the solution of the entire flow field. A self-similar solution of the linearized version of eqs. (6) (with $s = 0$) perturbed from the uniform state behind the incident shock (i)

Table 1
Conparison of Sakurai's theory (1986) with experimental data (Henderson and Siegenthaler, 1980)

ξ	θ	χ		ϕ_1		ω'		λ	
		cal.	exp.	cal.	exp.	cal.	exp.	cal.	exp.
0.5	4°25′	23°38′	21°50′		63°24′		72°24′	65°43′	63°24′
	4°26′	23°37′	24°09′	61°59′		77°40′	79°12′	63°34′	61°14′
	7°31′	20°32′	20°58′	61°214′			78°18′	65°04′	61°13′
					61°13′				
0.8	4°03′	17°07′	16°20′	69°50′	69°37′	69°10′	89°10′	74°38′	–
0.9	4°30′	11°20′	12°48′		72°42′		83°04′	83°32′	72°42′
	4°51′	11°00′	8°11′	74°10′	76°58′	88°50′	88°33′	88°30′	–
	8°30′	8°02′	7°20′		73°09′		85°27′	84°01′	75°00′

(Lighthill, 1948) is utilized for this purpose. It is found after some algebra that the Mach shock angle ϕ_3 at the triple point O′ is determined as

$$\lambda \equiv \pi - \phi_3 = \phi_1 + \left(\frac{1}{\sqrt{1-k^2}} - 1 \right) \theta.$$

Numerical values of χ, ϕ_1, ω', λ computed for given (γ, ξ, θ)-values are compared with corresponding experimental data (Henderson and Siegenthaler, 1980) in table 1 above.

Furthermore, the differences Δp and $\Delta\delta$ in the pressure and flow directions between the two flows immediately behind the reflected and Mach shocks at the triple point are derived from the solution, and are

$$\frac{\Delta P}{P'} = -\frac{2}{1-\xi}\left(\xi + \frac{\gamma-1}{\gamma+1} \right) \left\{ 1 - \sqrt{\frac{\gamma+1}{2\gamma}(1-\xi)} \right\} \theta,$$

$$\Delta\delta = \left\{ \frac{1}{\sqrt{\frac{\gamma+1}{2\gamma}(1-\xi)}} - 1 \right\} \theta.$$

These provide a modification of the Neumann condition,

$$\Delta p = \Delta\delta = 0,$$

which is seen to become inadequate when the shock gets weaker, that is, as $\xi \to 1$.

To obtain more accurate results, especially for the dependence of ϕ_1, ω', on θ, a more refined approximation to the system of self-similar equations (6) (with $s = 0$) is needed. This has already been done, at least partially, by the use of singular perturbation technique (Obermeier, 1984). It should be suggested that more accurate solutions can possibly be obtained by the numerical integration of the self-similar equations (6) with $s = 0$ or even by the full system of eqs. (6) itself.

4. Conclusions

1. The von Neumann theory is invalid for unsteady weak Mach reflection because the flow downstream of the reflected wave and the Mach shock is non-uniform (and subsonic), contradicting one of the assumptions of the theory. The nonuniformity is evidenced by the finite curvature of the waves near, and at, the triple point, and by the sensitivity of the flow in

this region to infinitesimal variations on the right-hand sides of eqs. (1) and (2). The finite curvature of the Mach shock observed in recent experiments and in numerical simulations implies that the thickness of the reflected wave may not be negligible.

2. While the reflected wave may be of significant thickness, it must nevertheless obey the Rankine-Hugonoit jump conditions. This is supported by the numerical work of Colella and Henderson, who imposed the R–H conditions on the reflected wave while allowing it to be arbitrarily thick. They obtained good agreement with experiment.

3. Viscous effects are only dominant when the incident shock is very near the corner in terms of viscous length. Walenta's experiments showed that this occurred for the first 100 to 150 mean free path lengths from the corner. This conclusion is also supported by Sakurai's transformation of the Navier–Stokes equations. Further away, viscosity effects appear to be at most of secondary importance and mainly due to the wall boundary layer.

4. The von Neumann theory is valid and successful for both steady and nonsteady flow when the incident shock is strong provided the triple point path is corrected, because the flow downstream of the reflected wave is then supersonic and uniform, as assumed by the theory.

5. The fundamental reason for the von Neumann paradox is the non-uniformity in the (subsonic) downstream flow in weak Mach reflection, and the uniformity in the (supersonic) downstream flow in strong Mach reflection.

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